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Does the Inertia of a Body Depend on its Energy Content?

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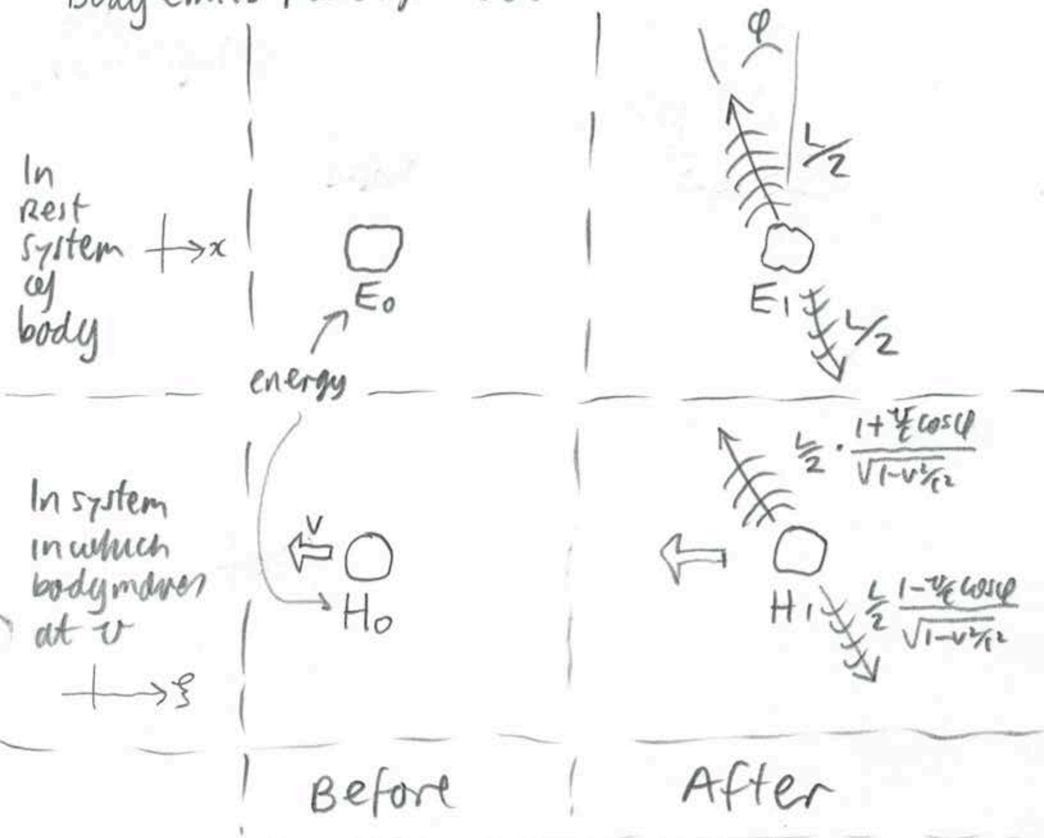
Goal: show: Body gains/ loses energy E $\Longrightarrow$ Body gains/ loses mass E/c^2 "Inertia of energy"

Body emits plane light waves



$$E_0 = E_1 + L$$

$$H_0 = H_1 + \frac{L}{\sqrt{1-v^2/c^2}}$$

Change in kinetic energy due to emission

$$= \underbrace{(H_0-E_0)}_{\text{kinetic energy before}} - \underbrace{(H_1-E_1)}_{\text{kinetic energy after}} = \underbrace{L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)}_{\frac{1}{2}\left(\frac{L}{c^2}\right)v^2 \text{ for small } v}$$

$\frac{1}{2}\underbrace{\left(\frac{L}{c^2}\right)}v^2$ for small v $\leftarrow$ Newtonian kinetic energy

$\rightarrow$ interpret as loss of mass L/c^2

see my companion article ↓

much simpler derivation uses momentum balance, momentum $= m\underline{v}$ } Precluded since AE's longitudinal, transverse mass ruled out mv as conserved momentum

Eliminating the reference to the Newtonian Limit: Rest mass of a body moving at v with kinetic energy $L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$

Rest mass m DEFINED as

$$\overset{\text{Force}}{F} = m \frac{d\overset{\text{speed}}{v}}{dt}\Big|_{v=0}$$

works for – Longitudinal & transverse mass
– Planck's definition $F = \frac{d}{dt}(m(v)\,v)$

Recover F from expression for kinetic energy

$$\underbrace{\text{"}dE = F.ds\text{"}}_{\text{works only for Planck definition of force ?}} \Rightarrow \frac{dE}{dt} = Fv \Rightarrow F = \frac{1}{v}\frac{dE}{dt}$$

or works at $v=0$ only for other definition??

$$E = L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) = L\left(\cancel{1}+\frac{1}{2}\frac{v^2}{c^2}+\frac{3}{8}\frac{v^4}{c^4}+\frac{5}{16}\frac{v^6}{c^6}\cdots\cancel{-1}\right)$$

Don't have to use series. Can also differential $\frac{1}{\sqrt{1-v^2/c^2}}$ directly. Just a little more work.

$$F = \frac{1}{v}\frac{dE}{dt} = \frac{1}{v}\cdot L\left(\frac{v}{c^2}\frac{dv}{dt}+\frac{3}{2}\frac{v^3}{c^4}\frac{dv}{dt}+\cdots\right)$$

$$= L\left(\frac{1}{c^2}\frac{dv}{dt}+\frac{3}{2}\frac{v^2}{c^4}\frac{dv}{dt}+\cdots\right)$$

At $v=0$ $\quad F = \frac{L}{c^2}\frac{dv}{dt} \Longrightarrow \boxed{m = L/c^2}$

OR ... Take shortcut:

For small v/c, Relativistic dynamics $\approx$ Newtonian dynamics

$$\lim_{v/c\to 0} m_{\text{relativistic}} = m_{\text{Newtonian}}$$

i.e. Let Newtonian mechanics do the job of introducing the definition of mass

Kinetic energy $L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) = \underbrace{\frac{1}{2}\frac{L}{c^2}v^2}_{\text{Newtonian kinetic energy with mass } L/c^2} - \frac{1}{8}\frac{L}{c^4}v^4 + \cdots$

$$\boxed{\frac{1}{\sqrt{1-v^2/c^2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \frac{5}{16}\frac{v^6}{c^6} + \cdots}$$

the easy way:

$f(x) = \frac{1}{\sqrt{1-x}} = (1-x)^{-\frac{1}{2}}$ $\qquad f(0) = 1$

$f'(x) = \frac{1}{2}(1-x)^{-\frac{3}{2}}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = \frac{3}{2.2.}(1-x)^{-\frac{5}{2}}$ $\qquad f''(0) = \frac{3}{2.2}$

$f'''(x) = \frac{3.5}{2.2.2}(1-x)^{-\frac{7}{2}}$ $\qquad f'''(0) = \frac{3.5}{2.2.2}$

$\vdots \qquad\qquad \vdots$

$$(1-x)^{-\frac{1}{2}} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{1}{2!}\cdot\frac{3}{2.2}x^2 + \frac{1}{3!}\frac{3.5}{2.2.2}x^3 + \cdots$$

set $x = v^2/c^2$

$$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \underbrace{\frac{1}{2!}\,\frac{3}{2.2.}}_{\frac{3}{8}}\,\frac{v^4}{c^4} + \underbrace{\frac{1}{3!}\,\frac{3.5}{2.2.2.}}_{\frac{5}{16}}\,\frac{v^6}{c^6} + \cdots$$

Newtonian Analog of Result

Define mass m as $\overset{\text{Force}}{F} = m \frac{dv}{dt}$ ← speed.

Kinetic energy "$dE = F.ds$" $\Longrightarrow \frac{dE}{dt} = F.V = m\frac{dv}{dt}v = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$

so kinetic energy $E = \frac{1}{2}mv^2 + \underbrace{\text{constant}}_{0 \text{ sin } E(0)=0}$

Hence, if kinetic energy is $\frac{1}{2} ? v^2$

read off that ? is mass.